

# Teaching statistics with technology

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## Introduction

The most recent *Australian Curriculum: Mathematics* document (ACARA, 2013) calls for students to develop the ability to use technology in their mathematical and statistical studies. Due to this increasing importance of integrating digital technologies in mathematics education, the national professional standards for teachers in Australia (AITSL, 2014) expect teacher education graduates to demonstrate technological, pedagogical and content knowledge (TPACK) in mathematics teaching and learning.

The Technological Pedagogical Content Knowledge (TPACK) conceptual framework for teaching mathematics, developed by Mishra and Koehler (2006), emphasises the importance of developing integrated and interdependent understanding of three primary forms of knowledge: technology, pedagogy, and content. TPACK relates to "...knowledge of the existence, components and capabilities of various technologies as they are used in teaching and learning settings, and conversely, knowing how teaching might change as a result of using particular technologies...". (Mishra & Koehler, 2006, p. 1028)

The TPACK conceptual framework is based upon the premise that effective technology integration for pedagogy around specific subject matter requires understanding of the dynamic relationship between all three knowledge components. Thus, teacher ICT training cannot be treated as context-free, but should be accompanied with emphasis on how technology relates to the pedagogy and content. The aim is to move teachers beyond techno-centric strategies that focus on the technology rather than the learning (Harris, Mishra, & Koehler, 2009). TPACK involves four components:

- (a) technological content knowledge about how to teach a subject with technology;
- (b) instructional strategies and representations;
- (c) students' thinking with technology; and
- (d) curriculum materials that integrate technology.

Niess and co-authors (2009) have reorganised these four aspects as central to teaching mathematics and have recommended standards and indicators for mathematics teachers' TPACK. TPACK has been embraced as a theoretical construct for effectively integrating technology in teacher professional development programs (Angeli & Valanides, 2009), for investigating the ways in which teachers acquire TPACK of mathematics (Cox & Graham, 2009), and for designing the implementation of Australian pre-service teacher education courses concerning to the development of pre-service teachers' TPACK capabilities in mathematics education (Larkin, Jamieson-Proctor & Finger, 2012).

The purpose of this article is to discuss the TPACK required for teaching Statistics, and to assist Australian teachers to make sense of the TPACK required for the effective integration of technology in statistics instruction.

## What does the Australian Curriculum say about the integration of technology in statistics instruction?

The information and communication technology (ICT) capability is one of the seven general capabilities as mentioned by ACARA (2013) where we can find the following:

Students develop ICT capability when they investigate, create and communicate mathematical ideas and concepts using fast, automated, interactive and multimodal technologies. They employ their ICT capability to perform calculations, draw graphs, collect, manage, analyse and interpret data; share and exchange information and ideas and investigate and model concepts and relationships.

In the *Australian Curriculum: Mathematics* (ACARA, 2013), digital technologies are integrated in classroom practices, as an integral part of teaching and learning of mathematics. The content of Statistics and probability requires young students from early primary schooling years to use digital technologies. In particular, the curriculum anticipates young Australian students will:

Collect data, organise into categories and create displays using lists, tables, picture graphs and simple column graphs, with and without the use of digital technologies. (ACMSP069)

The use of digital technologies is encouraged through the primary grades up to the secondary level while at the same time:

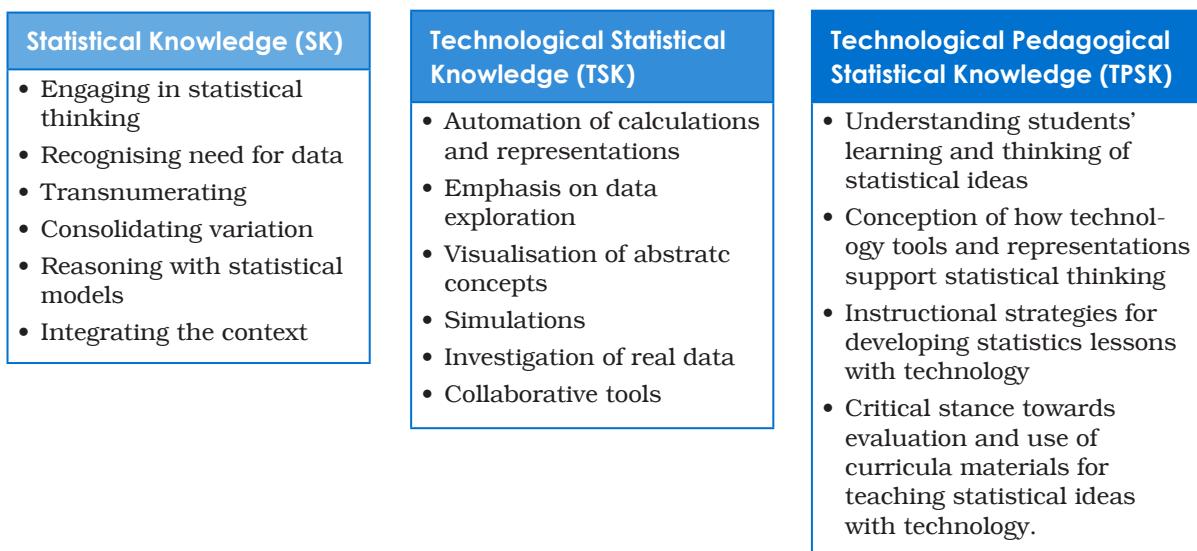
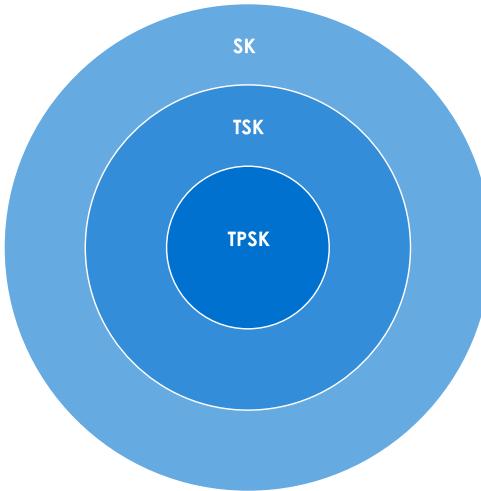
Students recognise and analyse data and draw inferences. They represent, summarise and interpret data and undertake purposeful investigations involving the collection and interpretation of data. They develop an increasingly sophisticated ability to critically evaluate chance and data concepts and make reasoned judgments and decisions, as well as building skills to critically evaluate statistical information and develop intuitions about data. (ACARA, 2013)

## What does the international research say about the need to teach statistics with technology?

When Lee and Hollebrands (2011) attempted to use the TPACK required for teaching statistics, they initially distinguished between two bodies of research on the use of technology in statistics. The first body of research involves studies that investigated the ways that teaching and learning of statistics have been influenced by technology. The second body of research involves studies that also included descriptions of the required technological and pedagogical content knowledge for the effective integration of technology in statistics instruction.

Lee and Hollebrands (2011) have characterised the important aspects of knowledge needed to teach statistics with technology, and have developed a Technological Pedagogical Statistical Knowledge (TPSK) framework (Figure 1) that integrates: (a) Statistical Knowledge (SK); (b) Technological Statistical Knowledge (TSK) about the use of technology to explore statistical ideas; and (c) Technological Pedagogical Statistical Knowledge (TPSK) of pedagogical issues related to teaching statistics.

TPACK and TPSK have been defined independently of each other by their authors, but we can see a strong relationship between the two. In particular, it makes sense to view TPSK—the knowledge needed for using technology when teaching statistics—as a special case of TPACK—the more general idea of content knowledge necessary for using technology for teaching (in any subject).



**Figure 1. Framework of Technological Pedagogical Statistical Knowledge (Lee, & Hollebrands, 2011, p. 362).**

In what follows, the notion of (TPSK) specifically for teachers of statistics is described.

### **Statistical Knowledge (SK)**

Lee and Hollebrands (2011) argued that teachers' SK is foundational for the development of their pedagogical knowledge, or of any other knowledge related to the use of technology in teaching statistics. Statistical thinking requires teachers to be engaged "as active learners and doers of statistical practices" (Lee & Hollebrands, 2011, p. 362), who have a well-developed understanding of "why and how statistical investigations are conducted and of the 'big ideas' that underlie statistical investigations" (Ben-Zvi & Garfield, 2004, p. 7).

### **Technological Statistical Knowledge (TSK)**

Within TSK, the focus is on teachers' familiarity with standard technologies in statistics education. It includes familiarity with technological developments such as *TinkerPlots* (Konold & Miller, 2005), whose dynamic features might support students' learning of statistics. In the same way the SK is foundational for development of pedagogical knowledge, knowledge of the technology used for statistical analysis and teaching statistics is a necessary foundation for any teacher's successful development of TPSK.

### Technological pedagogical statistical knowledge (TPSK)

The ultimate goal in the preparation of statistics teachers to use technology in teaching statistics is for them to develop TPSK, the specialised subset of pedagogical and content knowledge required to take advantage of technological tools for enhancing statistics instruction. Developing teachers' TSK and SK is an essential preliminary for teachers to develop specialised TPSK. Also required for development of TPSK are: (a) understanding of students' learning and thinking of statistical ideas when engaging with technology; (b) understanding of how technology tools and representations support statistical thinking; (c) knowledge of instructional strategies for technology-enhanced statistics lessons; and (d) according to Lee and Hollebrands (2011), a critical stance towards the evaluation and use of curricula materials for teaching statistical ideas with technology.

## What does Australian research literature say about the integration of technology in statistics instruction?

Several Australian academics used *TinkerPlots* dynamic data exploration software (Konold & Miller, 2005) that was designed specifically for students in upper primary and middle years of schooling. *TinkerPlots* gives students a dynamic and interactive learning environment in which they can construct graphs and interpret data and undertake purposeful investigations involving drawing inferences. The purpose of *TinkerPlots* is to provide a learning environment in which students have the opportunity to make sense of data, draw inferences and develop understanding of statistical concepts.

Watson and colleagues' (2011) book, entitled *Digging into Australian Data with Tinkerplots*, discussed how middle school students in Australia can use data sets collected in Australia for the purpose of exploring and analysing these data sets with *TinkerPlots* and developing statistical concepts when using features of the software. In essence, this publication by Watson and colleagues (2011) offers many examples of TPSK by providing engaging activities related to Australian datasets. Some examples of data collected include winners of the Melbourne cup, and data collected about Australian venomous snakes.

Prodromou (2011) reported on how Australian students who worked with *TinkerPlots* in 2009 and 2010 focused on making inferences about aspects. The following material uses a specific example to help reveal the aspects and character of TPSK in the context of a teaching episode using *TinkerPlots*.

The specific example is coming from a student-administered survey from Years 7 to 11 that was conducted by Prodromou in 2005 with the intent to examine students' exploration of the dataset when using *TinkerPlots* (Konold & Miller, 2005). The survey gathered information about students' weight and weight of students' backpacks<sup>1</sup>. Afterwards, the weight of a student's backpack was divided by the student's weight, and the calculated percentages were compared with the doctor's recommendations.

Teachers teach students how to use *TinkerPlots* to analyse the data collected from the student-administered survey, aiming to explore the variation of the means of the students' backpacks in different grades, which in fact are the samples drawn from the same population (ACMSP293). They use information from features of these samples, such as mean, proportions, and measures of spread, to predict characteristics of the population. The ultimate goal is to scaffold students to understand that larger samples better represent the population.

We explain in the following sections how this explicit example can demonstrate the TPSK required for teaching statistics. This may be of assistance when determining in what ways Australian teachers can make sense of the technological pedagogical content knowledge required for the effective integration of *TinkerPlots* in statistics instruction.

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1. The activity was inspired by a report written by students at Hermantown Academy and used by Prodromou (2011)

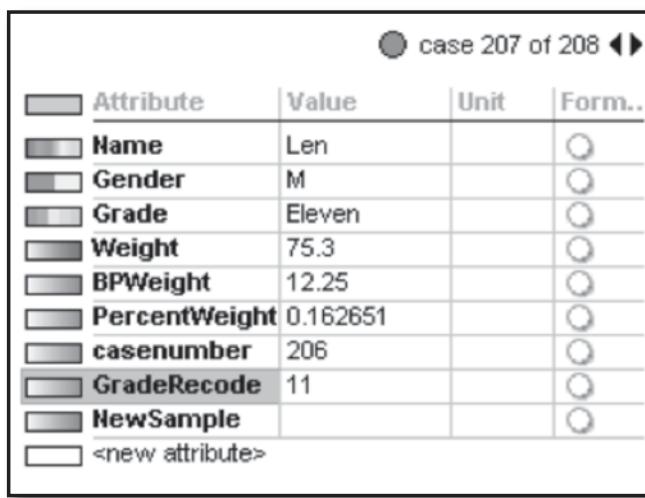
In the following section, we examine the Statistical Knowledge (SK), and the Technological Statistical Knowledge (TSK) that teachers need to have in order to develop Technological Pedagogical Statistical Knowledge (TPSK).

### Statistical Knowledge (SK)

In the example that follows, the necessary statistical knowledge involves the use of the data collected by the student-administered survey to explore the variation of means and proportions of random samples drawn from the same population (ACMSP293). For example, they must encourage students to look at the data before the data will be recorded in a stacked data card system in *TinkerPlots* and consider variation of data within the each Year and between different Years, considering the data collected from each year as samples drawn from the same school (population).

### Technological Statistical Knowledge (TSK)

In order to teach this example with *TinkerPlots*, the teachers need to know how to use the stacked data and system for the organisation of case-based data. Information about the attributes of each individual student is presented on a single card (Figure 2). Information about the attributes of each individual student is presented on a single card and this is one case in a dataset (Figure 2). Once the first card in the dataset is organised with the selected attributes and the units of measures, the remaining cards of all individual students will display the actual values of the attributes.



The screenshot shows a data card from the TinkerPlots software. At the top, it says 'case 207 of 208' with navigation arrows. The card is a table with four columns: 'Attribute', 'Value', 'Unit', and 'Form..'. The attributes listed are Name, Gender, Grade, Weight, BPWeight, PercentWeight, casenumber, GradeRecode, NewSample, and <new attribute>. Each attribute has a corresponding value and unit, and a small circle icon in the 'Form..' column.

Attribute	Value	Unit	Form..
Name	Len		<input type="radio"/>
Gender	M		<input type="radio"/>
Grade	Eleven		<input type="radio"/>
Weight	75.3		<input type="radio"/>
BPWeight	12.25		<input type="radio"/>
PercentWeight	0.162651		<input type="radio"/>
casenumber	206		<input type="radio"/>
GradeRecode	11		<input type="radio"/>
NewSample			<input type="radio"/>
<new attribute>			

Figure 2. An example of a single *TinkerPlots* data card.

*TinkerPlots* provides a spreadsheet that includes all the cards from all students (Figure 3) which were imported directly into *TinkerPlots* from *MSExcel* spreadsheets. The data that are imported are organised automatically into both the data cards and the spreadsheets. The teacher shows the students how to construct appropriate graphical representations of backpack weight versus gender (Figure 4), and backpack weight versus grade (Figure 5) from the data. *TinkerPlots* allow teachers and students to simultaneously construct multiple representations that are meaningful to them by enabling them to maintain control of the construction and manipulation of the graphical representations.

	Name	Gender	Grade	Weight	BPWeight
1	Emma	F	Seven	42.18	7.71
2	Fiona	F	Seven	42.18	6.35
3	Brenda	F	Seven	43.09	5.44
4	Elisa	F	Seven	43.54	7.26
5	Isabella	F	Seven	44	5.44
6	Jessica	F	Seven	44	6.35
7	Kathleen	F	Seven	44.45	5.9
8	Christina	F	Seven	44.45	5.44
9	Elizabeth	F	Seven	44.45	4.99
10	Heather	F	Seven	44.91	5.44
11	Jade	F	Seven	45.36	5.44
12	Julia	F	Seven	45.36	6.35
13	Keri	F	Seven	45.81	5.9
14	Gem	F	Seven	46.72	6.8
15	Kylie	F	Seven	47.17	8.16
16	Wendy	F	Seven	47.17	6.8
17	Chelsea	F	Seven	47.63	6.35
18	Susanne	F	Seven	48.08	6.8
19	Leah	F	Seven	48.08	7.26
20	Diana	F	Seven	48.99	7.71

Figure 3. Spreadsheet including all the cards.

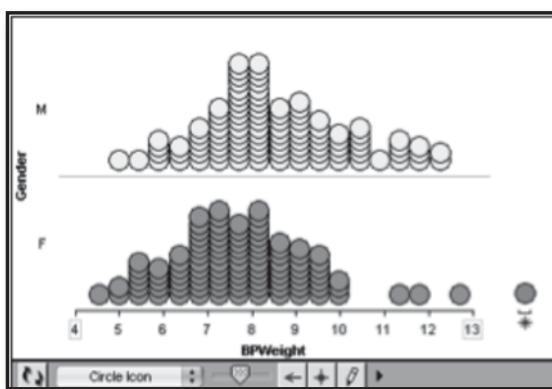


Figure 4. Backpack weight carried by females and males.

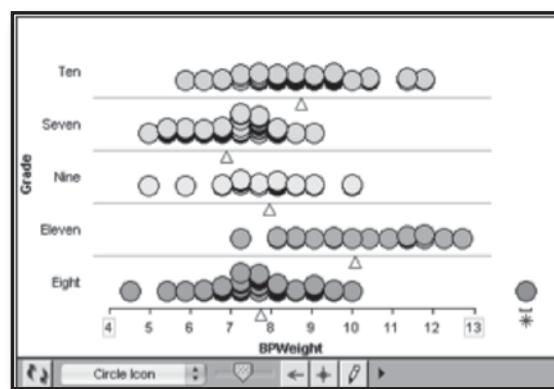


Figure 5. Backpack weight carried by 7-11 graders.

In this example, *TinkerPlots* provides students with the ability to create graphical representations that enable them to explore the distribution of the data and try to identify an apparent pattern. In particular, *TinkerPlots* allows teachers to simply drag an attribute into both the horizontal and vertical axes of the “Plot” window. For example, by simply dragging the attribute “grade” into the vertical axes and the attribute “backpack weight” into the horizontal axes, a graphical representation that displays backpack weight for each grade (sample) is created, allowing students to explore the variation of the means of the weights of backpacks for grades 7-11 (Figure 5) and compare the spread of the distribution of backpack weight. *TinkerPlots* allows teachers and users to take advantage of the vast capabilities of

*TinkerPlots* to automate computations of measures such as the backpack weight means for the different grades and organise a variety of graphical representations that would support students' visualisation when comparing the samples using sample properties to predict characteristics of the population.

In this example, in addition to allowing teachers and students to construct graphical representations, *TinkerPlots* also enhance students' ability to understand that larger samples better represent the population, by allowing students to create sliders and filters (Figure 6) that control the increase and decrease of the sample size (Prodromou, 2011). Sliders and filters are features of *TinkerPlots* that can be implemented to allow students to control the sample size that they select from the dataset.

*TinkerPlots* allows students to observe how the animated plots under study vary when the sample size slider adds or removes cases (data points) from the graph. For example, Figure 6 shows that up to Year 8 data will be included. If the slider points to 9, it means that Year 7 data, Year 8 data and Year 9 data will be included .

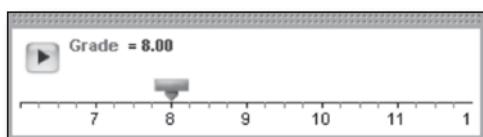


Figure 6. A slider for grade.

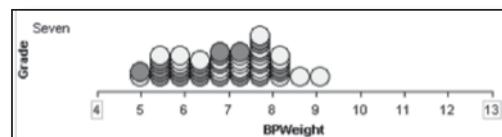


Figure 7. Backpack weight carried by grade 7 students.

According to Prodromou (2011), the sliders in *TinkerPlots* support the design of activities to evolve around the idea of growing samples starting from a sample that includes all the data collected from Grade 7 students (Figure 7), moving to a sample that includes all the data collected from Grades 7 and 8 (Figure 8), then all the data collected from grades 7, 8 and 9 (Figure 9), and all the data collected from Grades 7, 8, 9 and 10 (Figure 10) and finally including the entire population (Figure 11). Using a sequence of "growing sample" activities with the help of *TinkerPlots* is a pedagogical design that can help students visualise how larger samples better represent the population. The engagement of students with a sequence of "growing samples" activities helps students to visualise stable patterns generated by larger samples, and their larger samples are less variable and better represent a population. Asking students to make conjectures about the growing samples promotes students' reasoning about sampling in the context of variability and distribution. Such an approach is helpful in supporting coherent reasoning, based on the integration of key statistical concepts such as sampling, data, distribution, variability, and tendency.

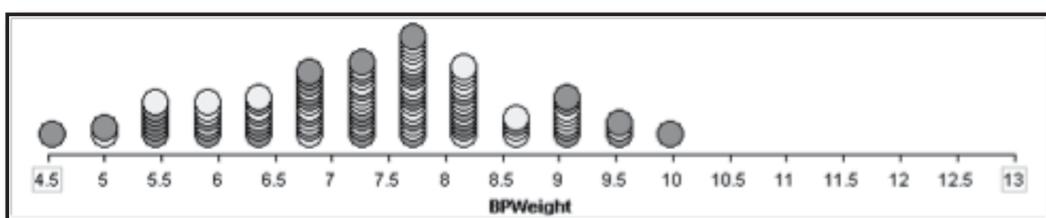


Figure 8. Backpack weight carried by grade 7 and 8 students.

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2. It is noteworthy to point out that although the scale is marked automatically in quarters by the software, when the slider is moved changes in the animated plots are made only when the slider points to 7, 8, 9, 10 or 11.

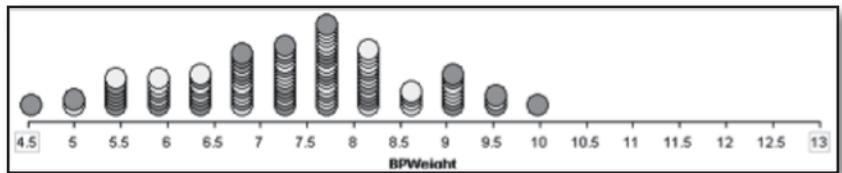


Figure 9. Backpack weight carried by grade 7, 8 and 9 students.

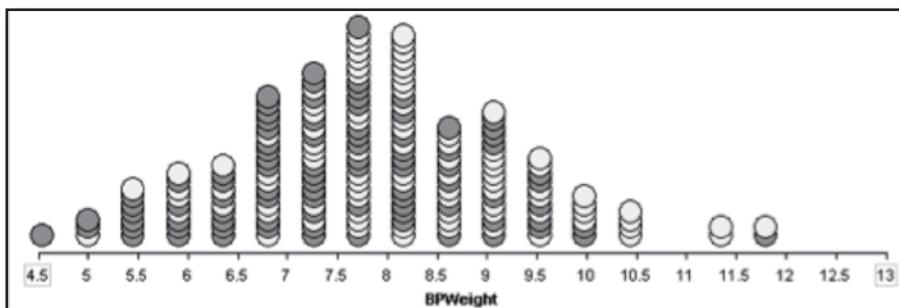


Figure 10. Backpack weight carried by grade 7, 8, 9 and 10 students.

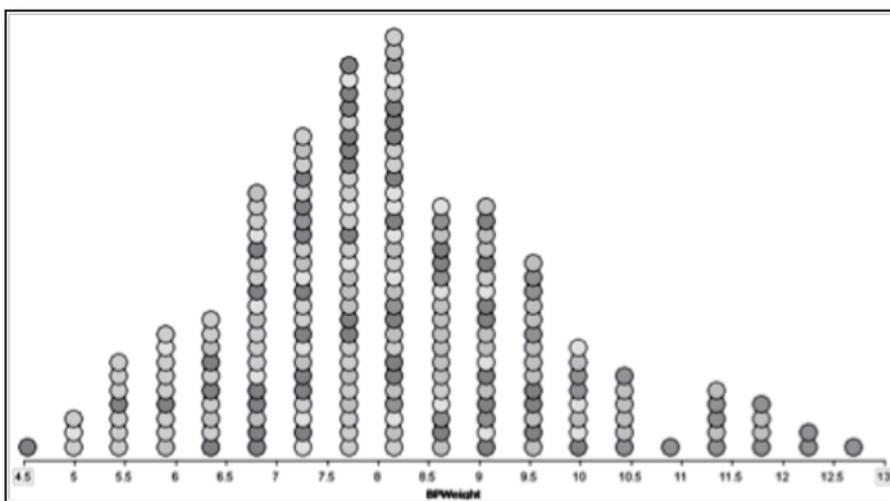


Figure 11. Backpack weight carried by grade 7–10 students.

### Technological Pedagogical Statistical Knowledge (TPSK)

In this example, teachers develop TPSK, capitalising on the technological affordances of *TinkerPlots* that provide a learning environment that facilitates the display of animated plots that enhance the instructional strategies for developing statistical lessons about “samples” and “sampling”. Developing teachers’ TPSK was essential to resort to research literature to gain a better understanding of how students understand that larger samples better represent the population when having students engage with *TinkerPlots*. When teachers use the sequence of the “growing sample” approach as a pedagogical design in their technology-enhanced statistics lessons, it helps students progressively develop their inferential reasoning about samples. Teachers must be aware of how students’ statistical learning and thinking developed, as discussed in Prodromou’s (2011) analyses of students’ reflections while working with similar activities. These analyses shed some light on the developmental process of students’ inferential reasoning about samples and sampling issues.

These findings from Prodromou’s study (2011) demonstrated that students developed new connections about the interplay of sample size and population as they manipulated the actual sample size with sliders in *TinkerPlots*. The animated visual representations of different sample size encouraged them to further link those concepts to other statistical

fundamental concepts during their investigations, such as spread, distribution, (explained) variation in data, unexplained variation, uncertainty, randomness and graph interpretation.

Students felt competent to explore the impact of the sample size on data representations when they engaged in data investigations which involved activities of growing and reducing the size of sample.

It is evidence that teachers develop TPSK about samples and sampling when they use the findings of similar studies as starting points for their professional practices in the classroom.

## Conclusion

Interactive technological environments such as *TinkerPlots* allow students to observe dynamic graphical representations of data that support their understanding of statistical concepts. In this example, teachers can teach students how to manipulate sample sizes and concurrently observe the constructed visual representations. Such meaningful technological environments reinforce reflection, which supports teachers in more effectively teaching statistics, and students in drawing better connections between problem-solving activities in statistics and real life situations. Dynamic educational software applications, in particular, have opened new potentials in statistics teaching and learning. We presented here an example of technology-enhanced activities we use in our professional development programs to acquaint teachers with dynamic software, as a means of discussing the different dimensions of TPSK.

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